

Q^2 -evolution of parton densities at small x values. Effective scale for combined H1&ZEUS F_2 data.

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We use the Bessel-inspired behavior of the structure function F_2 at small x , obtained for a flat initial condition in the DGLAP evolution equations. We fix the scale of the coupling constant, which eliminates the singular part of anomalous dimensions at the next-to-leading order of approximation. The approach together with the “frozen” and analytic modifications of the strong coupling constant is used to study the precise combined H1&ZEUS data for the structure function F_2 published recently.

A standard way to study the x behavior of quarks and gluons is to compare the data with the numerical solution to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [1] by fitting the parameters of x -profile of partons at some initial Q_0^2 and the energy scale Λ of Quantum Chromodynamics (QCD) [2, 3]. However, for the purpose of analyzing exclusively the small- x region, there is an alternative to carry out a simpler analysis by using some of the existing analytical solutions to DGLAP equations in the small- x limit [4, 5].

A reasonable agreement between H1 and ZEUS data [6] and the next-to-leading-order (NLO) approximation of perturbative QCD has been observed for $Q^2 \geq 2 \text{ GeV}^2$ (see reviews in [7] and references therein), which gives us a reason to believe that perturbative QCD is capable of describing the evolution of the structure function (SF) F_2 down to very low Q^2 values, where all the strong interactions are conventionally considered to be soft processes.

Recently, the ZEUS and H1 Collaborations have presented the new precise combined data [8] for the SF F_2 . In the recent paper [9], we analyzed the combined H1&ZEUS data based on the predictions obtained by using the so-called generalized doubled asymptotic scaling (DAS) approach [5].

At rather low Q^2 values, perturbation theory becomes to be marginal and some resummation is very helpful. Fortunately, at low x values the magnitude of the NLO corrections is strongly negative and effective increase of the scale of the strong coupling constant re-

duces the NLO corrections. Moreover, the higher values of the coupling constant scale leads to stabilization of perturbation theory at lower Q^2 values. Since in the framework of the generalized DAS approach [5], the singular and regular parts of anomalous dimensions contribute to different parts of parton distributions (PDs) it is natural to decrease (really even to neglect) the singular part of the NLO anomalous dimensions. Indeed, the singular parts contribute to the argument of the modified Bessel functions and determinates the Q^2 evolution at low Q^2 values.

The aim of this short paper is to continue the analysis in [9] by setting the new scale of the coupling constant, which eliminates the singular part of NLO anomalous dimensions.

I. In the generalized DAS approach [5] parton densities tend to some constant value at $x \rightarrow 0$ and at some initial value Q_0^2 . The main ingredients of the results [5], are:

- Both, the gluon and quark singlet densities are presented in terms of two components ("+" and "-") which are obtained from the analytic Q^2 -dependent expressions of the corresponding ("+" and "-") PD moments.
- The twist-two part of the "-" component is constant at small x at any values of Q^2 , whereas the one of the "+" component grows at $Q^2 \geq Q_0^2$ as

$$\sim e^\sigma, \quad \sigma = 2\sqrt{\left[|\hat{d}_+|s - \left(\hat{d}_{++} + |\hat{d}_+|\frac{\beta_1}{\beta_0}\right)p\right] \ln\left(\frac{1}{x}\right)}, \quad \rho = \frac{\sigma}{2\ln(1/x)}, \quad (1)$$

where σ and ρ are the generalized Ball-Forte variables,

$$s = \ln\left(\frac{a_s(Q_0^2)}{a_s(Q^2)}\right), \quad p = a_s(Q_0^2) - a_s(Q^2), \quad \hat{d}_+ = -\frac{12}{\beta_0}, \quad \hat{d}_{++} = \frac{412}{27\beta_0}. \quad (2)$$

Hereafter we use the notation $a_s = \alpha_s/(4\pi)$. The first two coefficients of the QCD β -function in the $\overline{\text{MS}}$ -scheme are $\beta_0 = 11 - (2/3)f$ and $\beta_1 = 102 - (38/3)f$ with f is being the number of active quark flavors. Note that the coupling constant $a_s(Q^2)$ is different at the leading-order (LO) and NLO approximations. Usually at the NLO level $\overline{\text{MS}}$ -scheme is used, so we apply $\Lambda = \Lambda_{\overline{\text{MS}}}$ below.

II. At low Q^2 , the value of the strong coupling constant is large. The most important terms, which are singular at $n \rightarrow 1$, where n is the Mellin moment, are collected in the result (1) for σ^2 . So, it is convenient to choose the scale μ^2 of the strong coupling at which the NLO contribution $\sim p$ in (1) vanishes. This choice is

$$\mu^2 = Q^2 \exp\left[\left(\hat{d}_{++} + |\hat{d}_+|\beta_1/\beta_0\right)/(|\hat{d}_+|\beta_0)\right] \approx 3.89Q^2 \quad (3)$$

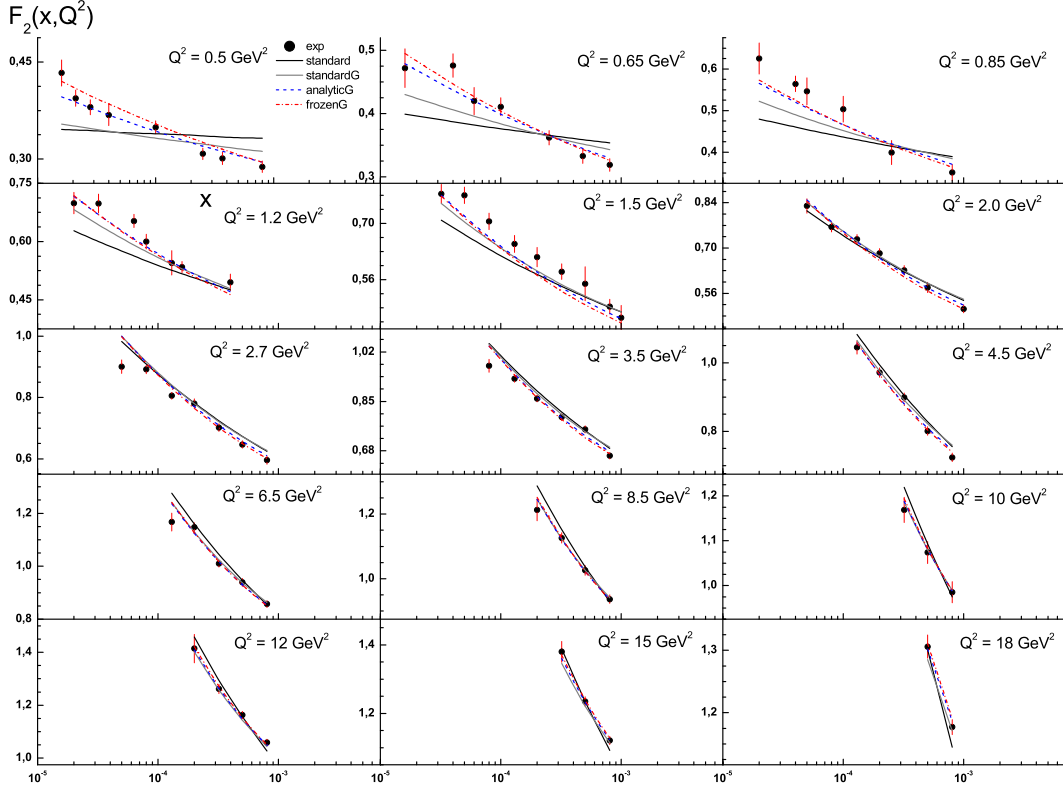


FIG. 1: x dependence of $F_2(x, Q^2)$ in bins of Q^2 . The combined experimental data from H1 and ZEUS Collaborations [8] are compared with the NLO fits for $Q^2 \geq 0.5 \text{ GeV}^2$ implemented with the scales of the strong coupling constant equal to Q^2 (solid lines) and to μ^2 (gray lines). The dot-dashed and dashed lines mark, respectively, the results based on the frozen and analytic versions of the strong coupling constant.

where the symbol \approx marks the case $f = 3$, which is relevant at low Q^2 values. We see that the choice (3) increases effectively the argument of coupling constant at low x values (see [10]).

In order to improve an agreement at lowest Q^2 values, the QCD coupling constant is modified in the infrared region. We consider two modifications. In the first case, which is more phenomenological, we introduce freezing of the coupling constant by changing its argument $Q^2 \rightarrow Q^2 + M_\rho^2$, where M_ρ is the ρ -meson mass (see [11] and discussions therein).

Thus, in the formulae (1) and (2) we have to carry out the following replacement:

$$a_s(Q^2) \rightarrow a_{\text{fr}}(Q^2) \equiv a_s(Q^2 + M_\rho^2) \quad (4)$$

The second possibility follows the Shirkov–Solovtsov idea [12] concerning the analyticity of the coupling constant that leads to additional power dependence of the latter. Then, in the formulae of the previous section the coupling constant $a_s(Q^2)$ should be replaced as follows:

$$a_{\text{an}}^{\text{LO}}(Q^2) = a_s^{\text{LO}}(Q^2) - \frac{1}{\beta_0} \frac{\Lambda_{\text{LO}}^2}{Q^2 - \Lambda_{\text{LO}}^2}, \quad a_{\text{an}}(Q^2) = a_s(Q^2) - \frac{1}{2\beta_0} \frac{\Lambda^2}{Q^2 - \Lambda^2} + \dots, \quad (5)$$

in the LO and NLO approximations, respectively. Here the symbol \dots stands for the terms that provide negligible contributions when $Q^2 \geq 1 \text{ GeV}$ [12].

III. By using the generalized DAS approach we have analyzed H1&ZEUS data for F_2 [8]. In order to keep the analysis as simple as possible, we fix $f = 4$ and $\alpha_s(M_Z^2) = 0.1168$ (i.e., $\Lambda^{(4)} = 284 \text{ MeV}$) in agreement with ZEUS results given in [6].

As can be seen from Fig. 1 the twist-two approximation is reasonable for $Q^2 \geq 2 \text{ GeV}^2$. At lower Q^2 , we observe that the choice (3) of the μ^2 scale provides only a little improvement in an agreement with data. However, the fits in the cases with “frozen” and analytic strong coupling constants, which are very close each other (see also [11, 13]), describe the data in the low Q^2 region significantly better than the standard fits with Q^2 and μ^2 scales. Nevertheless, for $Q^2 \leq 1.5 \text{ GeV}^2$ there is still some disagreement with the data, which needs to be additionally studied. In particular, the Balitsky–Fadin–Kuraev–Lipatov resummation [14] may be important here [15]. It can be added in the generalized DAS approach according to the discussion in Ref. [16].

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